Topology Optimization of Structures for Brittle/Ductile Fracture Resistance

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Frontiers in Computational Science and Engineering Research and Software
initiative for Computational Science and Engineering (iCSE)
Columbia University
May 12, 2021

COMPUTATIONAL MECHANICS
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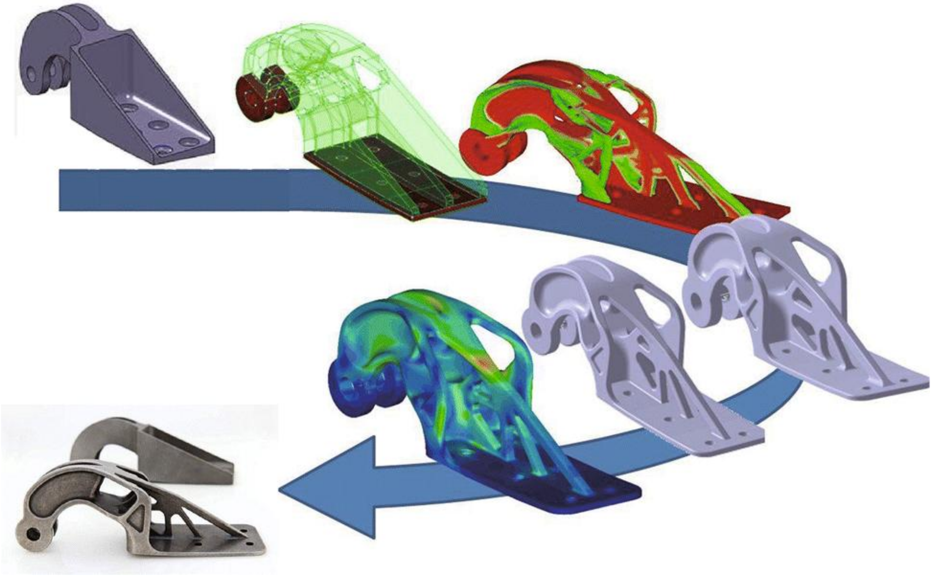
Structural Topology Optimization - 1

- Structural design optimization along with advancements in additive manufacturing enable engineering solutions with exceptional performance

Airbus A380 Wing Structures



Airbus A320 Nacelle Hinge Bracket

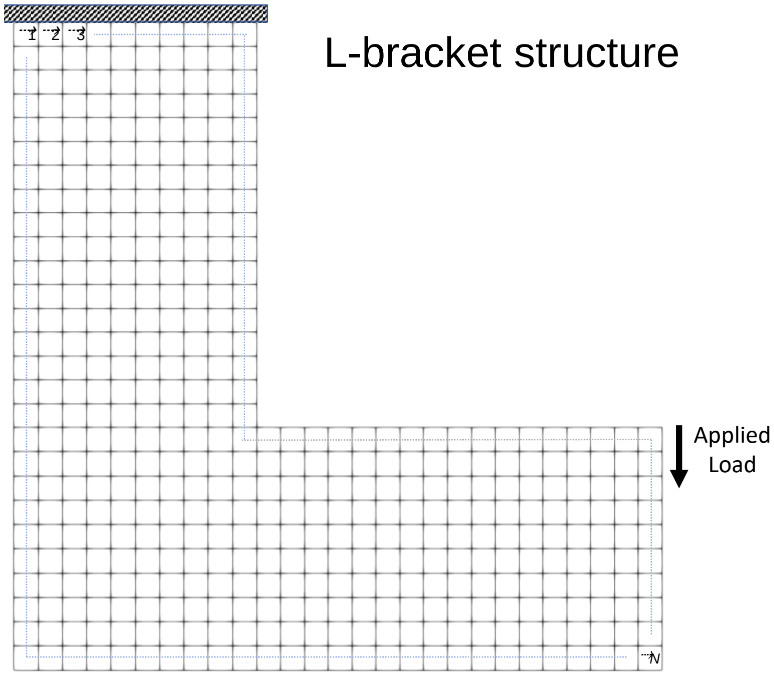
Krog et al., "Application of Topology, Sizing and Shape Optimization Methods to Optimal Design of Aircraft Components", Altair, 2011.
Tomlin M., Meyer J., "Proceeding of the 7th Altair CAE Technology Conference", 2011.

Structural Topology Optimization - 2

Path-*independent* topology optimization

- PDE-constrained optimization:

$$\begin{aligned}
\underset{\boldsymbol{\rho}=[\rho_1,\ldots,\rho_N]}{\text{minimize}} \quad & \overbrace{\frac{1}{V_{total}} \sum_{e=1}^{N_{elem}} \rho_e \, V_e}^{\text{Volume Fraction}} \\
\text{subject to} \quad & \boldsymbol{f}^T \boldsymbol{u} \leq C_{max} \\
& 0 \leq \rho_e \leq 1, \; e = 1, \ldots, N_{elem} \\
& \boldsymbol{R} = \boldsymbol{K}\boldsymbol{u} - \boldsymbol{f} = \boldsymbol{0}
\end{aligned}$$



L-bracket structure

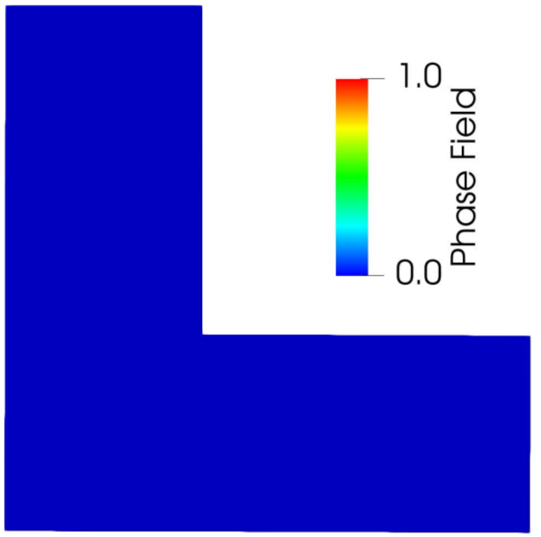
- SIMP Method (Solid Isotropic Material with Penalization)
- Adjoint method to compute sensitivities (typically requires one linear solve at every optimization step)

Bendsoe, M.P., Kikuchi, N., CMAME 1988

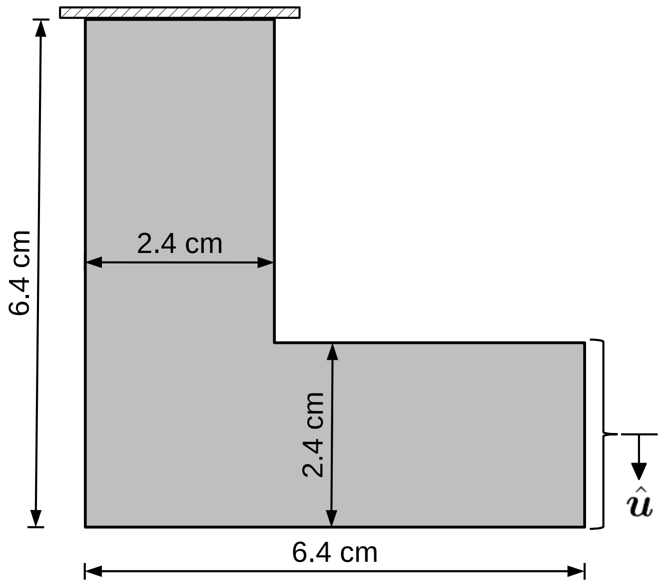
Topology Optimization for Brittle Fracture Resistance

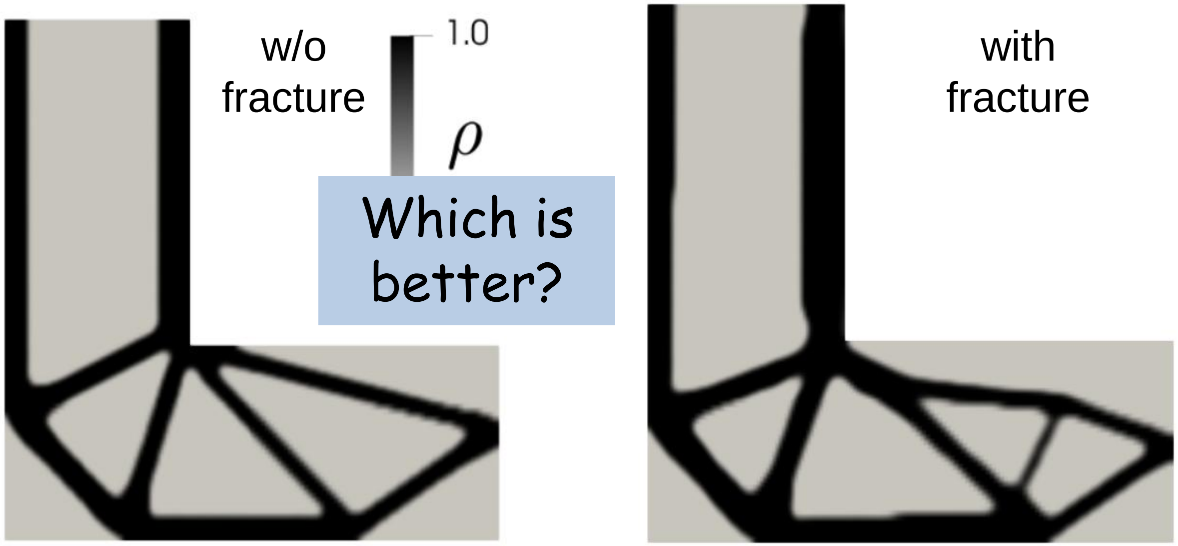
<u>Goal</u>: Achieve a minimal weight design with greater resistance to failure

- Two options to account for fracture resistance:
  - Constrain a stress/strain invariant
  - Include failure physics and constrain failure QOIs



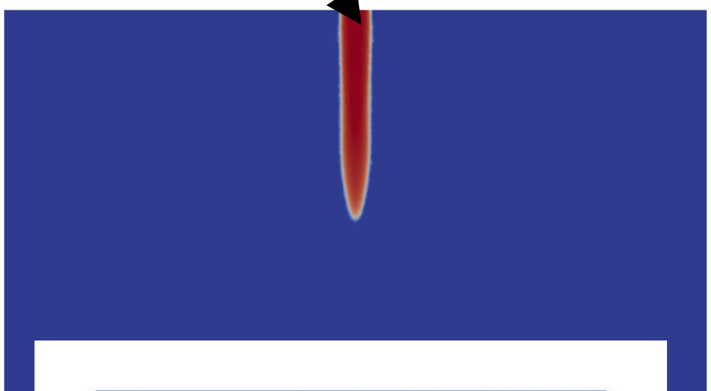
Example geometry





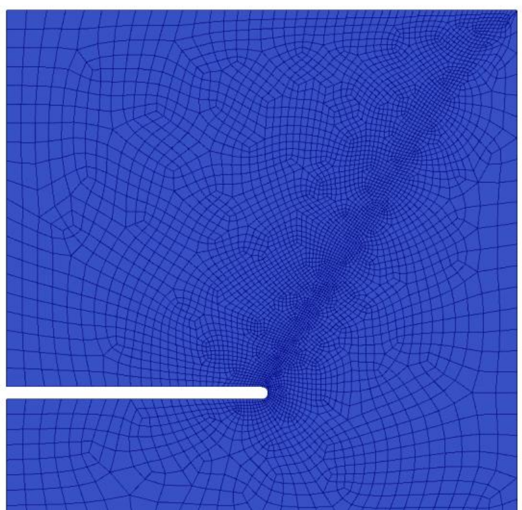
Discrete .vs. Continuum approaches to Fracture

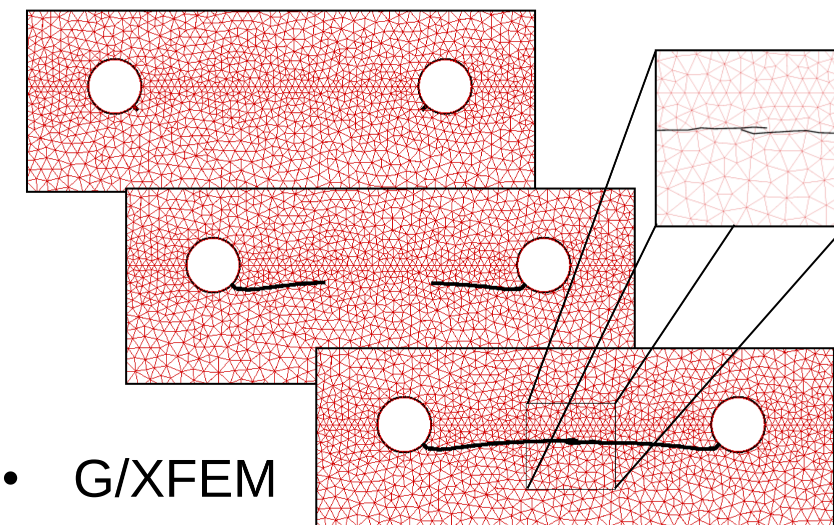
Continuum/Smeared Fracture:
damage mechanics



Discrete Fracture:
LEFM, Cohesive zone

- Nonloca
- Phase F



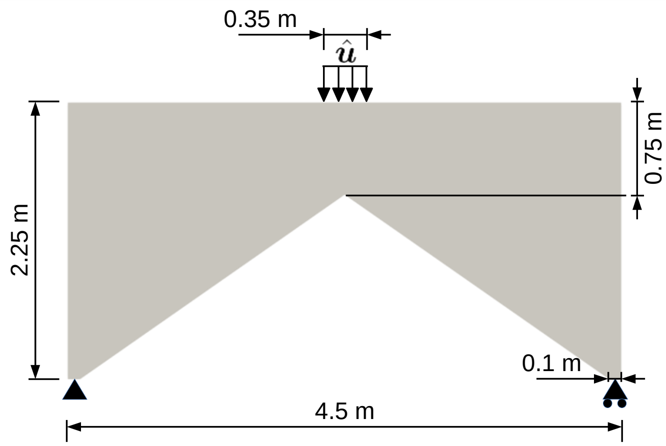


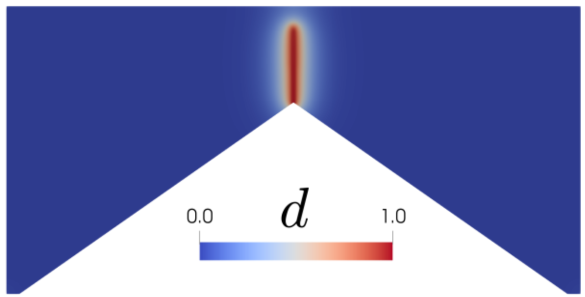
- G/XFEM

Topology Optimization for Brittle Fracture Resistance - 1

<u>Approach</u>: Include fracture physics and failure constraints

$$\begin{aligned} \underset{\rho_e,\ e=1,\ldots,N_{elem}}{\text{minimize}} \quad & VolumeFraction(\rho) \\ \text{subject to} \quad & \rho_{min} \leq \rho_e \leq 1,\ e = 1,\ldots,N_{elem} \\ & C_{min} \leq Compliance(\rho, u, d) \\ & FractureEnergy(\rho, d) \leq \hat{\psi}_f^{max} \\ & R^i = 0,\ i = 1,\ldots,N_{steps} \end{aligned}$$





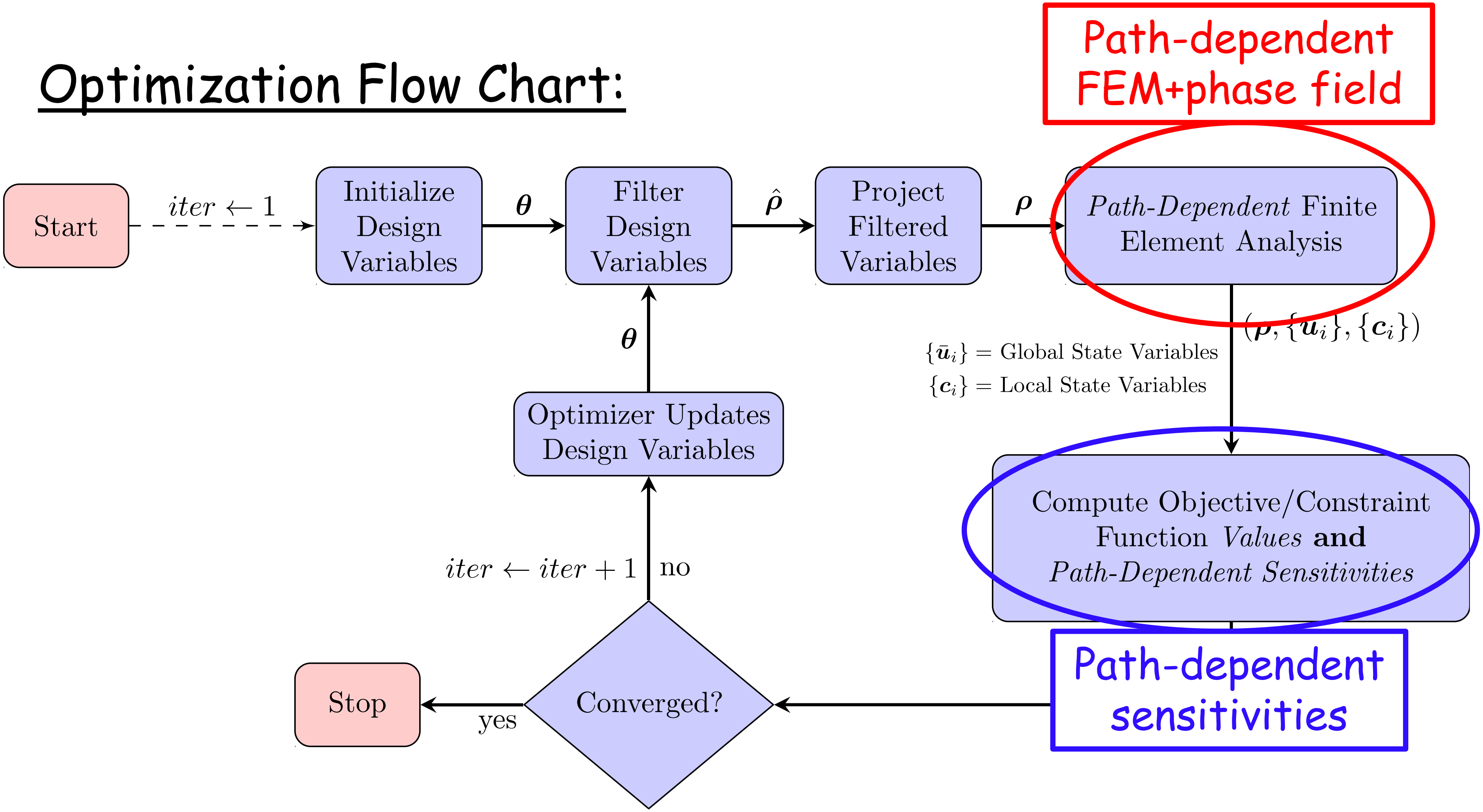
→ Path-*dependent* Topology Optimization

- Adjoint-based sensitivity analysis (transversed in time)
  - Efficient element level calculations

Russ, J.B., Waisman, H., Topology optimization for brittle fracture resistance, CMAME 2019

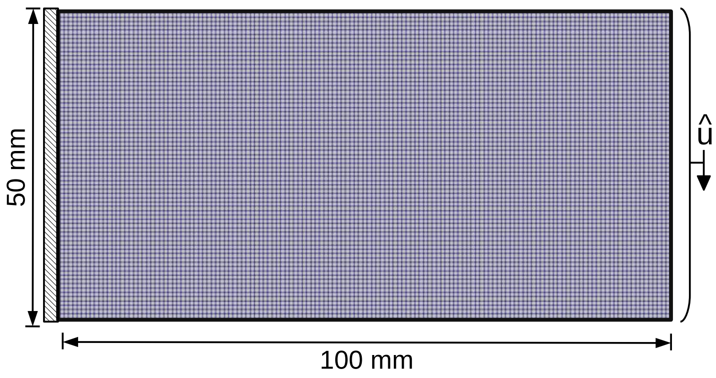
Path-*dependent* Topology Optimization

Optimization Flow Chart:



Topology Optimization for Brittle Fracture Resistance - 2

Cantilever Beam under bending



$$\frac{S}{W} = \frac{\text{Strength}}{\text{Weight}} = \frac{F_{max}}{\text{Mass}}$$

$$F_{max} = \text{Peak Load Capacity}$$

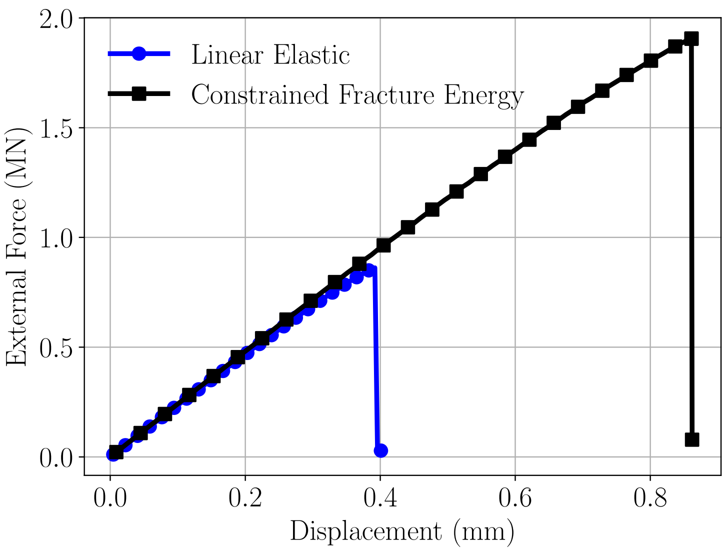
Benefit:

$121\%\ \frac{S}{W}$ increase

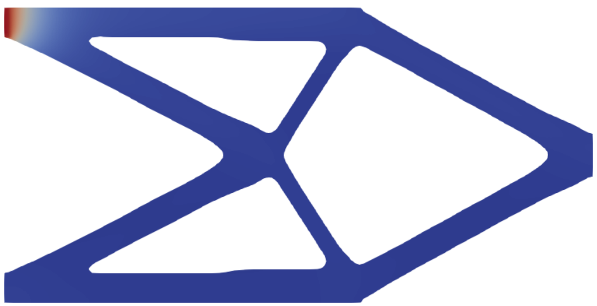
$75\%\ F_{max}$ increase

Cost:

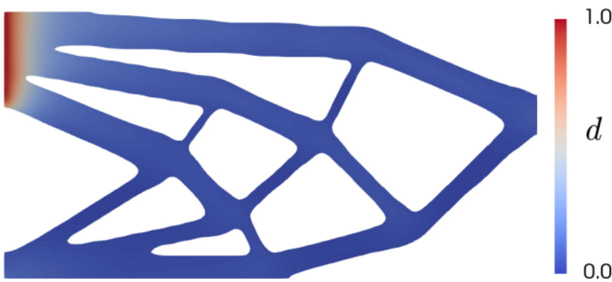
8.7% More Mass



Linear Elastic Design

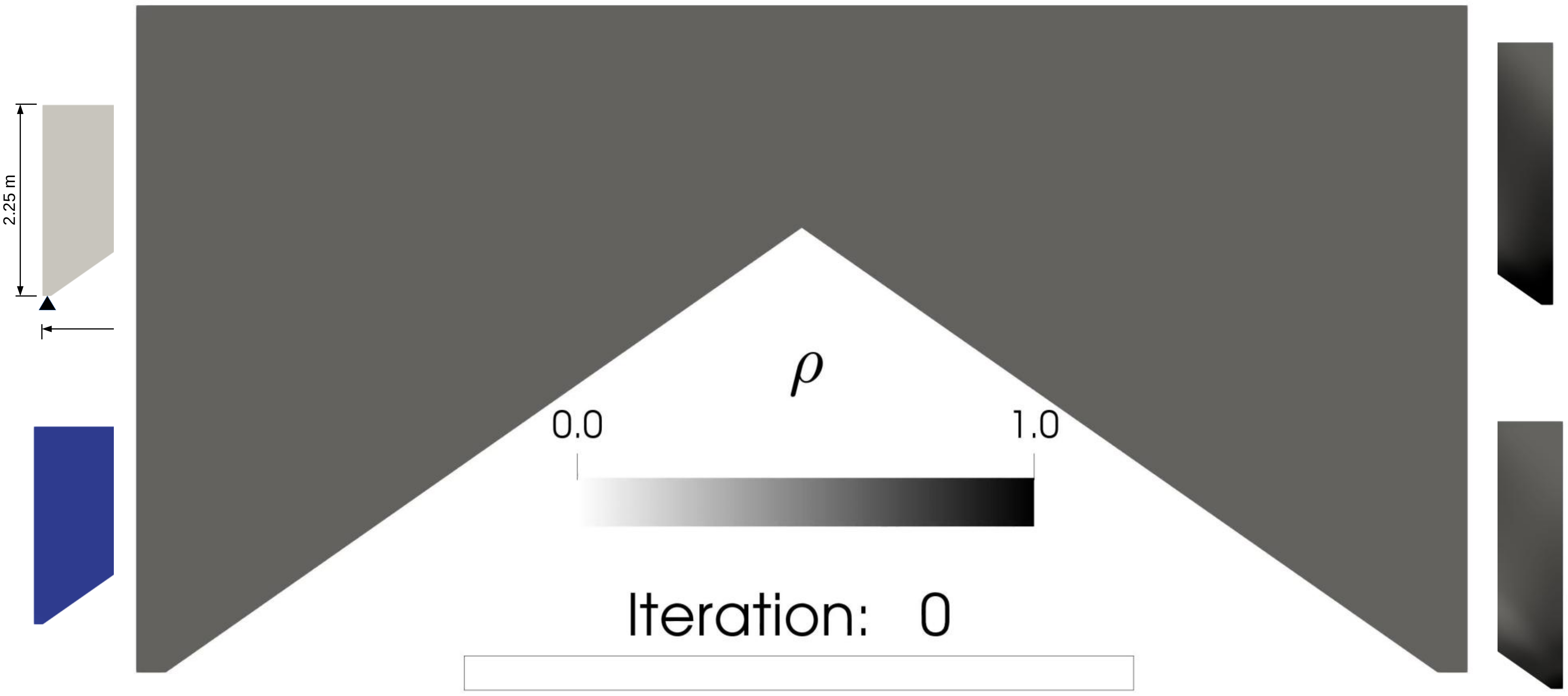


Constrained Fracture Energy Design



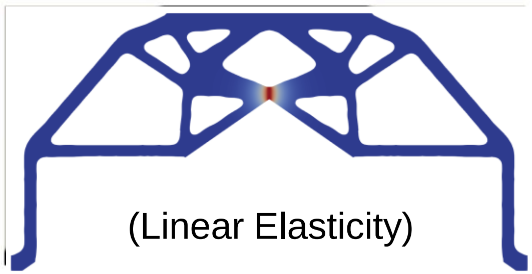
Russ, J.B., Waisman, H., Topology optimization for brittle fracture resistance, CMAME 2019

Topology Optimization for Brittle Fracture Resistance - 3

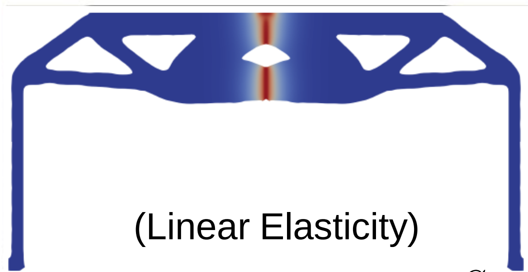


Russ, J.B., Waisman, H., A novel topology optimization formulation for enhancing fracture resistance with a single quasi-brittle material, IJNME 2020

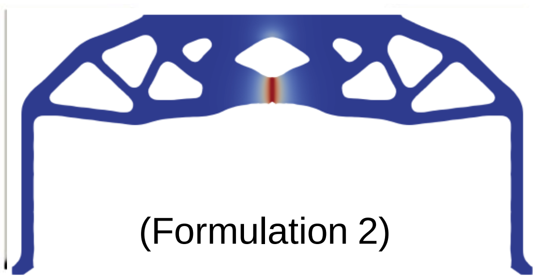
Topology Optimization for Brittle Fracture Resistance - 4



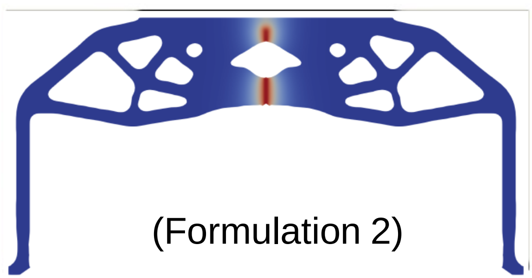
Work Only



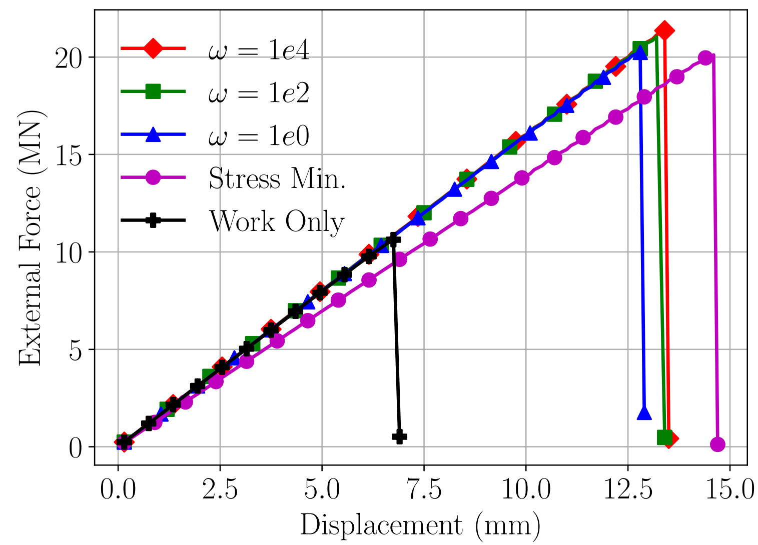
Stress Min.* $54.7\% \uparrow \frac{S}{W}$



$\omega = 1e0$ $65.8\% \uparrow \frac{S}{W}$



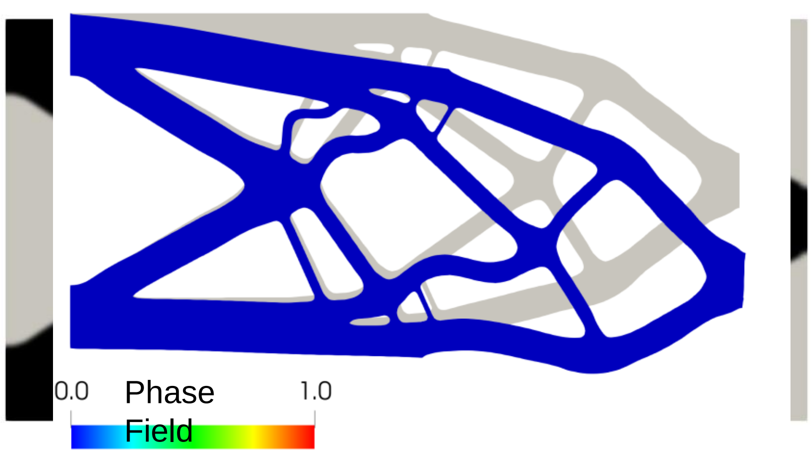
$\omega = 1e2$ $67.5\% \uparrow \frac{S}{W}$



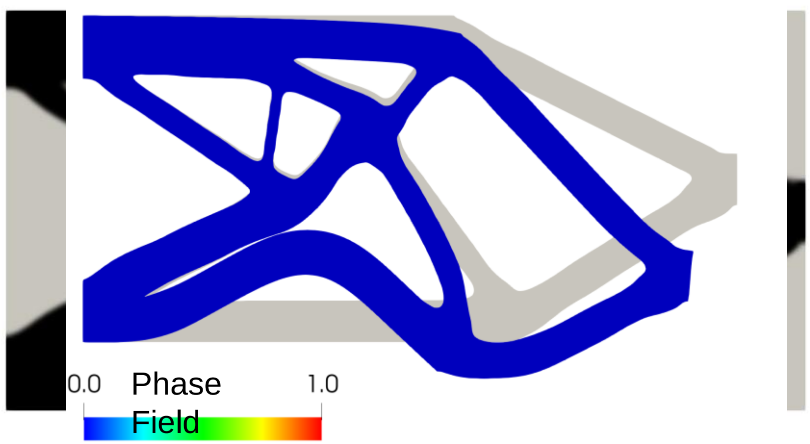
* Le et al, Stress-based topology optimization for continua. SMO, 2010

Topology Optimization for Ductile Failure and Buckling - 1

- **Goal:** A topology optimization formulation for increasing the *peak load capacity* of a structure, in addition to it's *structural toughness*
- Currently, no elastoplastic topology optimization formulation exists which incorporates *both* ductile failure and buckling resistance
  - ➢ Topology Optimization is performed in small strain framework, but Final topologies are evaluated with finite strain ductile Phase Field Model



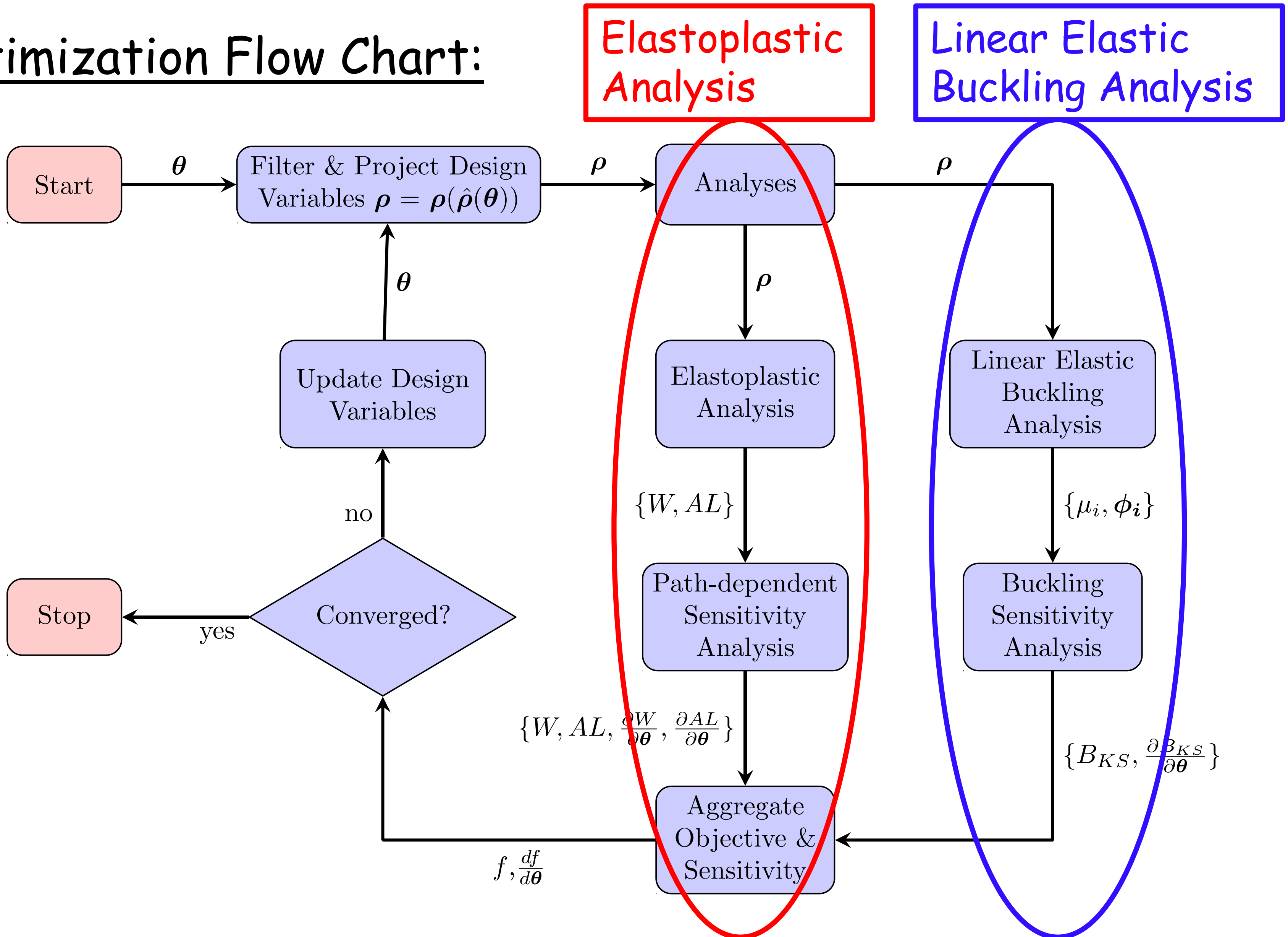
Maximize Total Work Only



Maximize Total Work With Ductile Failure Constraints

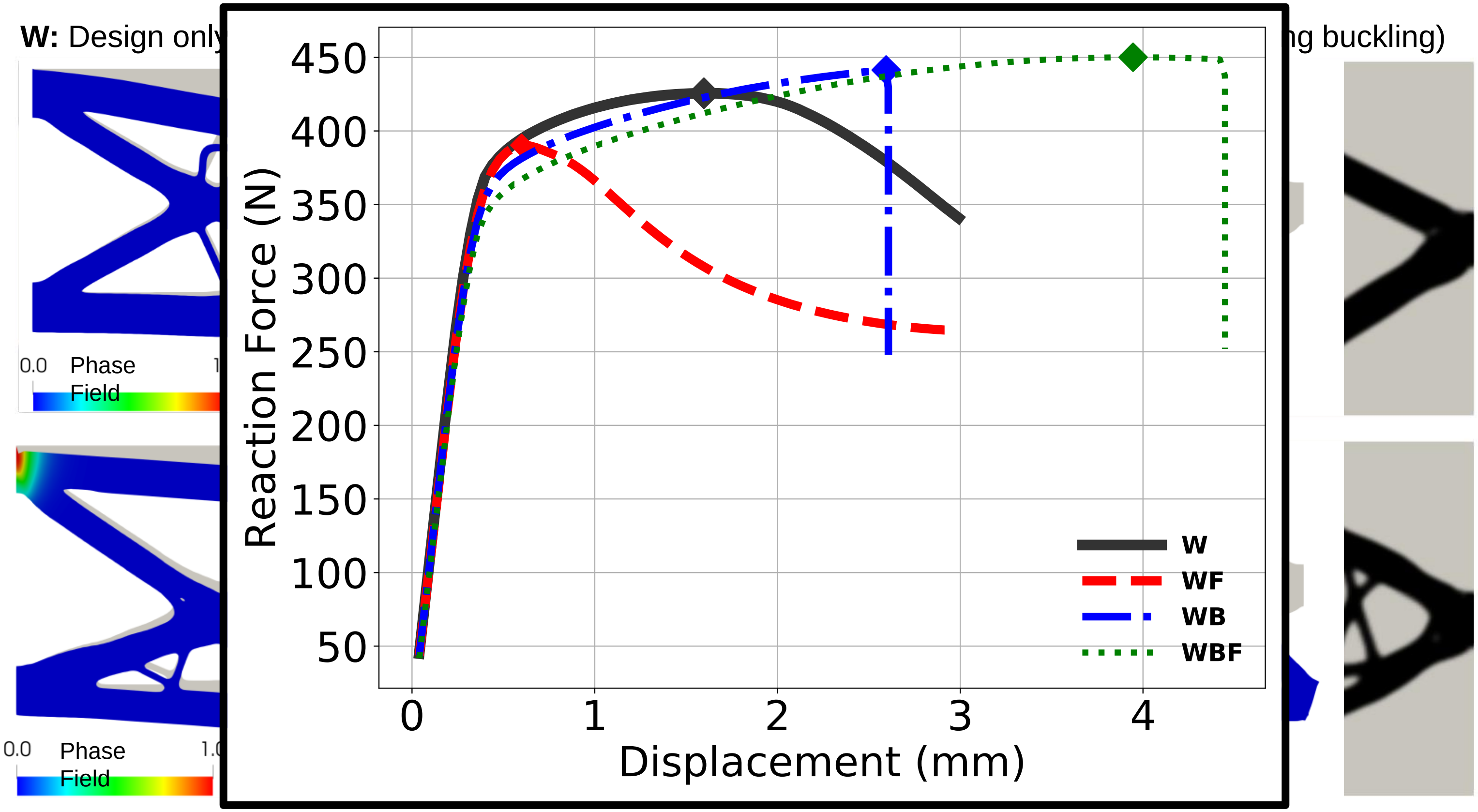
Russ, J.B., Waisman, H, A novel elastoplastic topology optimization formulation for enhanced failure resistance via local ductile failure constraints and linear buckling analysis, CMAME, 2021

Optimization Flow Chart:



Topology Optimization for Ductile Failure and Buckling - 2

W: Design only ... ng buckling)



WB: Design with buckling only (omitting ductile failure)

WBF: Design with ductile failure constraints and buckling

Russ, J.B., Waisman, H, A novel elastoplastic topology optimization formulation for enhanced failure resistance via local ductile failure constraints and linear buckling analysis, CMAME, 2021

Concluding Remarks

Many opportunities for original research on Design Optimization of Structures