A benchmark for the development of hybrid physics-data driven wall-layer models for turbulence

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Krayenhoff et al. (2020) Urban Climate

Research focus #1: Urban fluid mechanics

LES horizontally-averaged
LES @ tower location
Tower measurements
Hurricanes are the costliest natural catastrophe in the US.

More intense and destructive hurricanes will occur in the future due to the increase in the sea surface temperature.
Research focus #3: Machine learning for turbulence modeling

Figure 5 Block-diagram for the PINNs network we used to obtain the results.
Wall-layer modeling for large-eddy simulation

Cost DNS
\[ \sim \text{Re}^{2.5} \]

Bose & Park (2018)
A. Lozano-Duran and J. Bae (2021)

Cost wall-modeled LES
\[ \sim \text{Re}^{0–1} \]

Compute only the outer layer using a grid that does not resolve the inner layer, while modeling the transport of momentum and heat from the inner to the outer layer.
Introduction

Computational fluid dynamics for high Reynolds number flows

A. Lozano-Duran and J. Bae (2021)
- Commonly found in flows over swept wings and bow/stern regions of ships.
- Skewed 3D mean-velocity profile.
- Often created by (geometry-induced) cross-stream pressure gradient.
Introduction

Counter-intuitive behavior of 3-D unsteady boundary layers

Imposing additional strain may weaken the magnitude of Reynolds stress

On average, mean strain rate and Reynolds stress are misaligned

\[-\bar{u}_i u_j \neq \nu_t \bar{S}_{ij}\]

\[\bar{u}_i \theta \neq d_t \frac{\partial \theta}{\partial x_i}\]

Important for accurate prediction of friction coefficient, heat transfer

Theories and models established for equilibrium wall turbulence are typically not valid for unsteady 3-D flows (e.g., the law-of-the-wall, scaling laws, turbulence closures, reduced models…)

Wall-layer models models based on eddy viscosity can not predict these effects
Methodology

Idealized setup for the analysis

DNS of transient 3D channel flow with sudden spanwise pressure gradient (incompressible Newtonian fluid at constant density)

\[
\frac{dP}{dx} = 7.36 \\
\Delta_+^{y} = 0.35 \\
\Delta_+^{z} = 4.29
\]
DNS of transient 3D channel flow with sudden spanwise pressure gradient (incompressible Newtonian fluid at constant density)

Simulation campaign:

\[ \text{Re}_\tau \approx 180, \; 550, \; 950 \]

\[ \frac{dP}{dz} = 1, \ldots, 100 \]

DNS

\[ \Delta_x^+ = 7.36 \]
\[ \Delta_y^+ = 0.35 \]
\[ \Delta_z^+ = 4.29 \]
Methodology

Idealized setup for the analysis

\[
\frac{dP}{dx} \quad \frac{dP}{dz} = 10 \frac{dP}{dx}
\]

\[Re_x \approx 1000\]

*streamwise velocity at \( y^+ = 40 \)*
Results

Analysis of selected flow statistics

Streamwise mean velocity

Spanwise mean velocity

Tangential Reynolds stress

\[ Re_\tau \approx 180 \]

\[ \frac{dP}{dz} = 10 \frac{dP}{dx} \]
Flow regimes for different $\frac{dP}{dz}$ and $Re_\tau$

Non-equilibrium for all eddies: $\frac{dP}{dz} > \mathcal{O}(Re_\tau)$

Flow regimes
Results

Flow regimes for different $\frac{dP/dz}{dP/dx}$ and $Re_\tau$

Quasi-equilibrium for all eddies:
$$\frac{dP/dz}{dP/dx} < O(1)$$

Non-equilibrium for all eddies:
$$\frac{dP/dz}{dP/dx} > O(Re_\tau)$$

Flow regimes

$\text{quasi-equilibrium}$
Coherent structures type analysis and phenomenological model

1. Layer of incoherence flow diffusing vertically from the wall
2. Weakening of the streaks
Can existing LES wall models predict non-equilibrium 3D TBL?

\[ \text{Re}_r = 934 \quad (\text{Re}_{bulk} \approx 20000) \]
Wall-layer modeling for large-eddy simulation

- No grid resolution to resolve 3D profile
- Assumed that wall stress \( \propto \) mean flow

Invalid with mean flow three dimensionality (e.g., swept-wing BL, atmospheric BL)
Results

Wall-stress magnitude prediction in $Re_{\tau}=950$ 3D channel.

$Re_{\tau} = 934$

$$\frac{dP}{dz} = 10 \frac{dP}{dx}$$

LES grid: $\Delta x = 0.2h$, $\Delta z = 0.15h$, $\Delta y = 0.07h$ (uniform spacing)

($\Delta x^+ = 180$, $\Delta z^+ = 133$, $\Delta y^+ = 60$)
Wall-stress angle prediction in $Re_\tau=950$ 3D channel.

$$Re_\tau = 934 \quad \frac{dP}{dz} = 10 \frac{dP}{dx}$$

LES grid: $\Delta x = 0.2h$, $\Delta z = 0.15h$, $\Delta y = 0.07h$ (uniform spacing)

$(\Delta x^+ = 180$, $\Delta z^+ = 133$, $\Delta y^+ = 60)$
In the case of **Separated flow**

1) Some wall models, by construction, can represent only monotonically-varying velocity profiles (e.g. EQWM, algebraic wall-stress models).

2) These models, when driven by the LES data away from the wall (e.g., 3rd point), can produce wall shear force with wrong directions.

\[
\frac{d}{dy}(\nu + \nu_t) \frac{du}{dy} = 0 \quad \rightarrow \quad \text{Either} \quad \frac{du}{dy} > 0 \quad \text{or} \quad \frac{du}{dy} < 0
\]

\[
\tau_w > 0 \quad \quad \quad \quad \quad \quad \quad \quad \tau_w < 0
\]
**Conclusions**

1. Generated a benchmark of cases to support the development and validation of wall-layer models for large-eddy simulations.
2. Provided an overview on one of the cases from a flow physics perspective.
3. Wall layer models show variable performance in the prediction of non-equilibrium effects and none is able to capture the observed stress reduction.

**Path forward:**

1. Leverage the database of benchmarks to develop hybrid physics-data driven wall-layer models that can be trained on the above dataset and that are able to capture non-equilibrium effects.

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*Giometto et al. 2018, CTR Annual Res. Briefs*  
*Lozano-Duran et al., 2020, J. Fluid Mech*  

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